Proceedings of the Eleventh (2014) Pacific/Asia Offshore Mechanics Symposium **www.isope.org**
Shanghai, China, October 12-16, 2014

ISBN 978–1 880653 90-6: ISSN 1946-004X

Numerical Simulations of 3D Liquid Sloshing Flows by MPS Method

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ABSTRACT: In this paper, a modified MPS (Moving Particle Semi-Implicit) method is applied into 3D liquid sloshing to show its capability in modeling 3D sloshing flows. Violent sloshing flow in 3D rectangular is investigated. Furthermore, impact pressure value obtained by 2D and 3D simulation is compared, which indicates that 3D MPS simulation is able to produce smoother pressure field and thus makes better agreement with experiment than 2D MPS simulation. In addition, the snapshots of free surface deformation and velocity field are compared between experiment, 2D and 3D MPS simulation. Results show that the details of free surface deformation, such as the breaking region of the wave and the splash of liquid fluid can be capsized by 3D MPS method effectively.

KEY WORDS: MPS method; 3D liquid sloshing; rectangular tank; sloshing; impact pressure.

INTRODUCTION

Sloshing refers to the movement of liquid inside a partially filled tank due to external excitations. The liquid sloshing in LNG (Liquefied Natural Gas) tank is of significant importance in marine industry. When the amplitude of the ship motion is very large or its frequency is close to the natural frequency of the liquid tank, violent sloshing flows may appear, exerting strong impact pressure on the wall of the tank, which may cause too large deformation on the structure locally, and affect the stability of ship globally. As a result, it's essential to predict the impact pressure accurately to avoid severe structural damage.

Due to the ever-increasing interests in liquid sloshing, a number of researchers have applied numerical simulation methods to the sloshing problem. Most of the numerical simulations are focused on grid-based methods, such as FDM (Finite Difference Method, Lee et al., 2008), FEM (Finite Element Method, Wang and Khoo, 2005), VOF (Volume of Fluid, Lee et al., 2007) and so on. In recent years, meshfree methods such as SPH (Smoothed Particle Hydrodynamics, Gingold and Monaghan, 1977) and MPS (Moving Particle Semi-Implicit, Koshizuka and Oka, 1996; Koshizuka et al., 1998) have been developed to model fluid motion with large deformation of free surface. In particle method, flow is modeled as an assembly of interacting particles which have physical properties, such as mass, momentum, and energy, etc. Since these particles have no fix topography among each other, meshless methods are more flexible to deal with the large deformed free surface flows (Zhang and Wan, 2011a).

There has been some work on liquid sloshing based on particle method, such as Cui and Liu (2009) applied SPH to simulate the sloshing phenomenon in a 2D tank subject to the motion of surge and pitch， showing a good agreement between numerical simulation and experiment in terms of free surfaces deformation, but overestimated impact pressure. Shao et al (2012) implied SPH method to model viscous incompressible liquid sloshing with different external excitations and different structures. The obtained numerical results including flow pattern, wave height, pressure field, and pressure load on solid walls were agreeable with experimental results. Delorme et al (2009) simulated 2D sloshing and discussed the influence of viscosity and of density re-initialization on the SPH results, good agreements were obtained in terms of free surface shape and global dynamics of the flow between experimental and SPH results. Khayyer and Gotoh (2011) simulated 2D violent sloshing based on improved MPS method, obtaining smooth impact pressure with small oscillation. Zhang and Wan (2012) computed liquid sloshing in 2D low-filling tank based on modified MPS method, in which good agreement between numerical results and experimental data is obtained.

Although the particle method can better deal with the aforementioned problems, it also has some drawbacks, such as spurious fluctuations in pressure value and high computation cost. To suppress the fluctuation in flow field, an improved MPS method has been adopted in this paper. The modified schemes include: a mixed source term in pressure Poisson equation (PPE), and a surface detection with a high accuracy. The mixed source term of PPE consists of two parts: the divergence free condition and the particle number density condition. While the surface detection method is based on the asymmetry of arrangement of neighbor particles, smooth pressure field is obtained based on the present MPS method (Zhang and Wan, 2012).

Because of the great amount of computation and higher requirement of computational stability in 3D MPS sloshing, fewer 3D liquid simulations are computed by MPS method than 2D simulations. Thus, we utilize the improved MPS method to simulate 2D and 3D liquid sloshing in a rectangular tank under translational excitation in this paper. Violent sloshing flow in 3D rectangular tank is investigated. To study the dimensional effect in MPS method, pressure histories obtained by 2D and 3D MPS are compared with experimental data. Furthermore, deformation of free surface and velocity field are also compared between 2D simulation, 3D simulation and experiment.

NUMERICAL SCHEME

Governing Equations

For incompressible, viscous flows, the governing equations are expressed in Lagrangian form as follows:

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla \cdot \boldsymbol{V} = 0 \tag{1}$$

$$\frac{D\boldsymbol{V}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{V} + \mathrm{g} \tag{2}$$

Where ρ denotes the density, P is the pressure, V is the velocity, g is the gravity acceleration and ν the kinematics viscosity. Because MPS method is based on Lagrangian approaches, the time differentiation is represented by substantial derivative involving advection terms in both of the equations. As a result, the numerical diffusion induced by advection terms can be eliminated.

Particle interaction models

In MPS method, the governing equations are transformed into the equations of particle interaction. The weight function in the particle interaction is defined as kernel function. Different from the kernel function in traditional MPS method (Koshizuka and Oka, 1996), an improved kernel function (Zhang and Wan, 2011b) without singualr is adopted to smooth the computational results in this paper:

$$W(r) = \begin{cases} \dfrac{r_e}{0.85r + 0.15r_e} - 1 & 0 \le r < r_e \\ 0 & r_e \le r \end{cases} \tag{3}$$

Where, $r = \left| r_j - r_i \right|$, denotes the distance between two particles, r_e radius of the influence area of each particle.

To calculate the weighted average in MPS method, particle number density is defined as (Masahiro Kondo, 2010):

$$\langle \mathrm{n} \rangle_i = \sum_{j \neq i} W\left(\left| r_i - r_j \right| \right) \tag{4}$$

This value is assumed to be proportional to the density, so the particle number density can be applied instead of density in particle discretization. As the density is almost constant in incompressibility calculation, we use the constant n^0 instead of n^i in the formulation of weighted average.

Gradient Model

In this paper, the gradient operator can be discretized into a local weighted average of radial function as follows:

$$<\nabla P>_i = \frac{D}{n^0}\sum_{j \neq i}\frac{P_j + P_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i)\cdot W(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{5}$$

where D is the number of space dimensions, r represents coordinate vector of fluid particle, $W(r)$ is the kernel function and n^0 denotes the initial particle number density for incompressible flow. Eq.5 can not only improve the stability of the calculations but also maintain the momentum conservation.

Laplacian Model

The Laplacian operator is formulated as (Koshizuka et al., 1998):

$$<\nabla^2\phi>_i = \frac{2D}{n^0\lambda}\sum_{j \neq i}(\phi_j - \phi_i)\cdot W(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{6}$$

The parameter λ is introduced as:

$$\lambda = \frac{\sum_{j \neq i} W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)\cdot|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}{\sum_{j \neq i} W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)} \tag{7}$$

In Eq.6 the parameter λ is introduced to keep the increase of variance equal to that of the analytical solution.

Model of incompressibility

In traditional MPS method, the incompressible condition is represented by keeping the particle number density constant. In each time step, there are two stages: first, temporal velocity of particles is calculated based on viscous and gravitational forces, and particles are moved according to the temporal velocity; second, pressure is implicitly calculated by solving a Poisson equation, and the velocity and position of particles are updated according to the obtained pressure.

Here we adopt a mixed source term for PPE proposed by Tanaka and Masunaga (2010), which combines the velocity divergence and the particle number density. This improved PPE is rewritten by Lee et al. (2011) as:

$$<\nabla^2 P^{\mathrm{k}+1}>_i = (1-\gamma)\frac{\rho}{\Delta t}\nabla\cdot V_i^* - \gamma\frac{\rho}{\Delta t^2}\frac{<n^k>_i - n^0}{n^0} \tag{8}$$

where： γ is a blending parameter (between 0.01 and 0.05) to account for the relative contributions of the two terms.

Free Surface boundary condition

In the MPS method, the free surface dynamic condition is enforced by assigning zero pressure for surface particles. By now, some approaches have been developed to detect the free surface particles. Koshizuka and Oka (1996) recognizes the surface particles according to the particle number density. Tanaka & Masunaga (2010) and Lee et al. (2011) judge the surface particle by using number of neighbor particles. Khayyer et al. (2011) proposed a new criteria based on asymmetry of neighboring particles in which particles are judged as surface particles according to the summation of x-coordinate or y-coordinate of particle distance. In the present study, we employ a detection method which is also based on the asymmetry arrangement of neighboring particles, but use different equations, aiming at describing the asymmetry more accurate, as follows (Zhang and Wan, 2011c):

$$< \boldsymbol{F} >_i = \frac{D}{n^0}\sum_{j\neq i}\frac{1}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}(\boldsymbol{r}_i-\boldsymbol{r}_j)W(\boldsymbol{r}_{ij}) \qquad (10)$$

Where, the vector function $\boldsymbol{F}$ represents the asymmetry of arrangements of neighbor particles.

Thus, particles satisfying:

$$<|\boldsymbol{F}|>_i \ > \ \alpha \qquad (11)$$

are considered as surface particle, where α is a parameter with a value of $0.9|\boldsymbol{F}|^0$ in this paper, $|\boldsymbol{F}|^0$ is the initial value of $|\boldsymbol{F}|$ for surface particle.

NUMERICAL SIMULATIONS

In this section, numerical tests have been conducted for both 2D and 3D liquid sloshing in rectangular tank under horizontal motion. Fig.1 shows the computational model, which is the same as the experimental model given by Chang. The length of the tank is L=0.79m, its height is H=0.48m, and its width is W=0.48m. The depth of water is d=0.144 m, corresponding filling level is 30%. Sloshing pressure at point P is measured in the simulation. Point P is near the free surface, 0.12m from the bottom of the tank.

The tank is subject to sinusoidal horizontal excitation:

$$x = -A\cos(\omega t) \qquad (12)$$

Where A is the amplitude of excitation with the value of 0.0575m, ω is excitation frequency, here $\omega = 4.49 rad/s$, which is equal to the first order resonant frequency of fluid motion.

In 2D sloshing simulation, 5944 particles are used, among which 4396 are fluid particles. The initial particle space is 0.005 m and the time step is $2\times10^{-4} s$. The acceleration of gravity is $\text{g}=9.81\text{m/s}^2$. The density of water is $\rho = 1000 \text{kg}/m^3$.

In 3D simulation, 663458 particles are used, among which 417620 are fluid particles. The value of other parameters, such as the initial particle, the initial particle space, the time step, the acceleration of gravity, the density of water and so on, is the same as the value in 2D simulation.

It is essential to note that, in this paper, in this paper, we only investigate the dimensional effect on the numerical results. Therefore, the initial particle space between 2D MPS and 3D MPS is the same. Effect of the total particle number will be discussed in other papers.

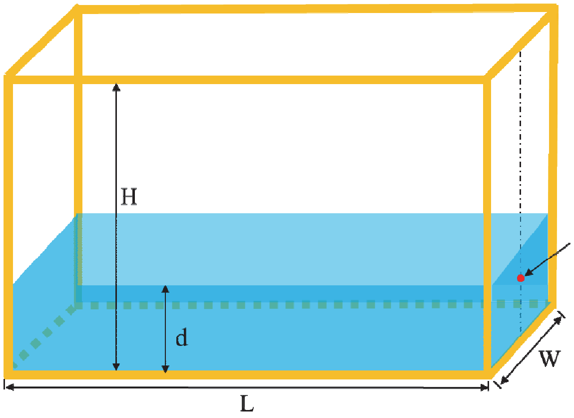
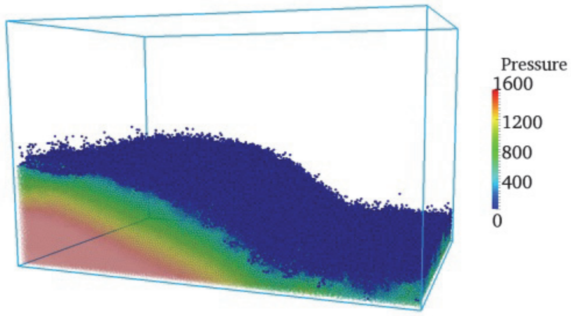
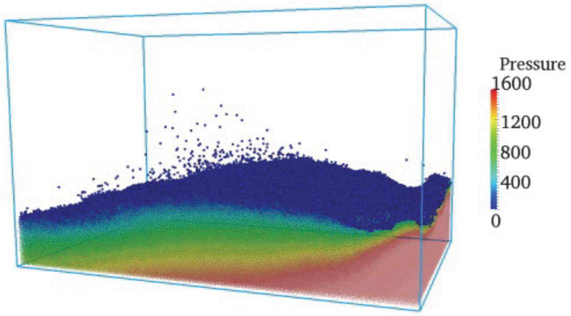


Fig. 1 Schematic of the tank

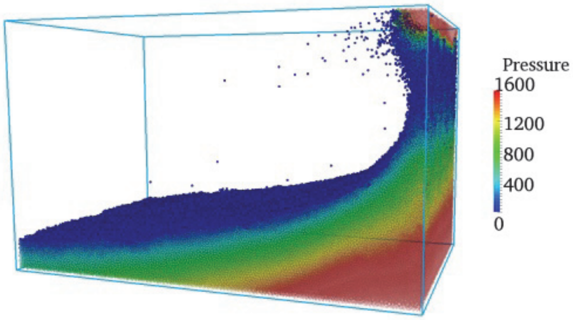
Snapshots of 3D MPS simulation in one period are shown in Fig.2. It can be seen that wave propagates in the tank and impacts on the side walls and ceiling of the tank. As the excitation frequency is equal to resonant frequency, flow is quite violent. Breaking wave and splash water are observed. When the tank reaches its maximum displacement and starts to return, liquid with large horizontal velocity impacts on the side wall and causes large impact pressure, seen in Fig. 2(b). Then an up-shooting jet is formed, which hits on the top of the tank, resulting in large impact pressure on the upper corner, see Fig. 2(c). After that, the jet breaks and falls down due to gravity. The falling liquid hits on the underlying liquid, and disturbs the free surface, as shown in Fig. 2(d). Though the flow is violent, the present MPS is capable of computing such complicated flows.



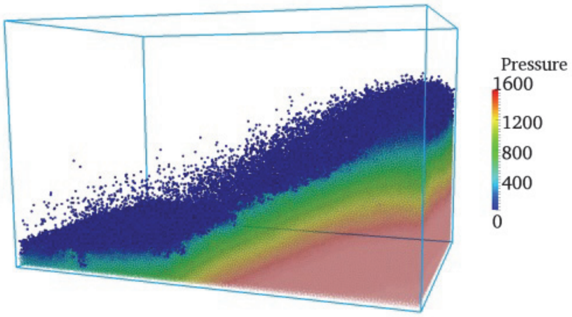
(a) $t = nT$



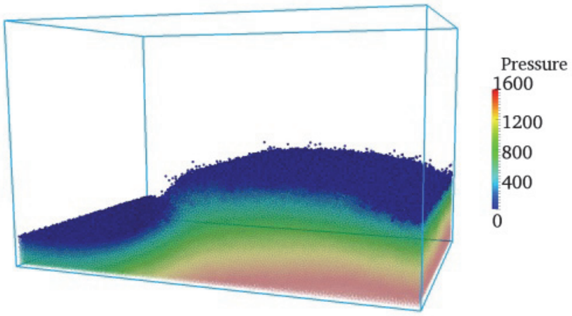
(b) $t = nT + 0.1T$



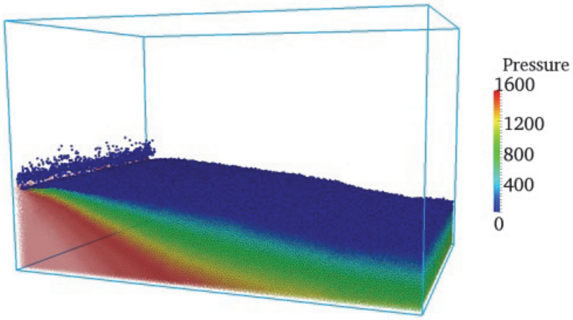
(c) $t = nT + 0.2T$



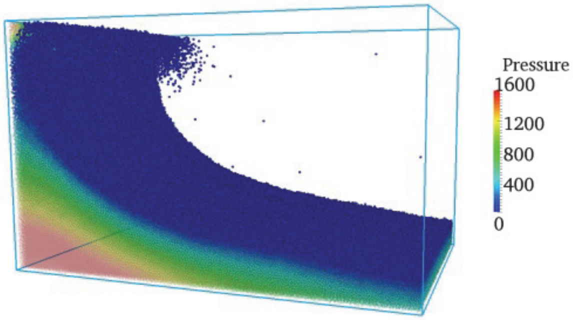
(d) $t = nT + 0.45T$



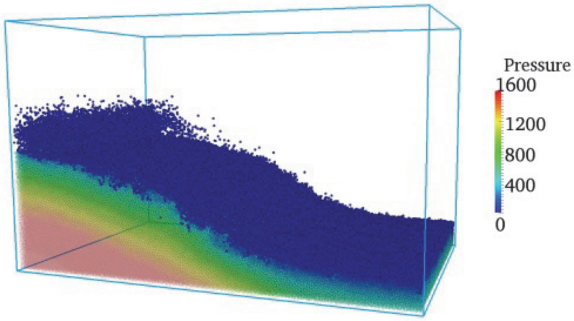
(e) $t = nT + 0.5T$



(f) $t = nT + 0.6T$



(g) $t = nT + 0.7T$



(h) $t = nT + 0.9T$

Fig. 2 Evolution of the sloshing waves

The time history of experimental and computed pressure at point P are compared in Fig. 3. As can be seen, the global shape of the pressure curve is well reproduced by both 2D and 3D MPS method, which indicates that both 2D and 3D MPS method can predict the impact pressure induced by liquid sloshing in general.

From the pressure history, we can also observe that the pressure pattern looks like a typical ‘‘church roof’’ profile. As is shown in Fig. 2(b), when the tank reaches its maximum position on the right, the momentum direction of the fluid suffers instantaneous change. As a result, the breaking wave impinges the side wall, causing an impulsive pressure with a high peak value at point P at $t = nT + 0.1T$. Then, the sloshing flow turns upward along the right-side tank wall, resulting relatively uniform pressure that lasts about 0.4T. Subsequently, the elevated sloshing liquid starts to break and falls from the top wall due to gravity, causing the second impact on the water lower which induces another peak value. After that the water moves toward the left-side wall, lowering the water depth near the right-side wall to below point P so the pressure is essentially zero.

Fig. 4 shows the details of impact pressure evolution at point P obtained by experiment, 2D MPS and 3D MPS method in one period. It can be seen that the evolution of pressure obtained by 3D simulation shows better agreement with experimental data. While the second peak of the pressure history in 2D simulation is slightly higher than experimental value, which may be caused by the unphysical oscillation in pressure field in 2D simulation. From the comparison, we can also observe that 3D simulation pressure curve is much smoother. This is because each particle has more neighbor particles in 3D computation, leading to the weighted average value smoother.

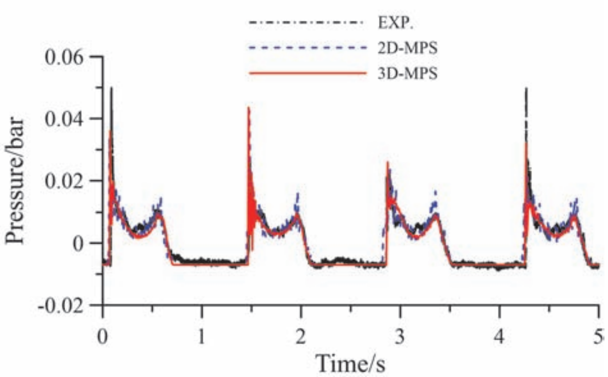


Fig. 3 Comparison of impact pressure at point P between experiment, 2D MPS and 3D MPS method

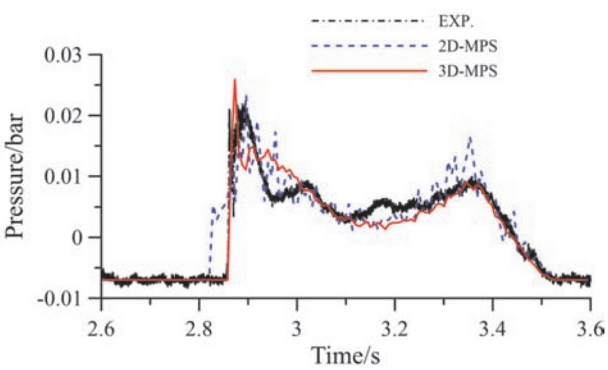
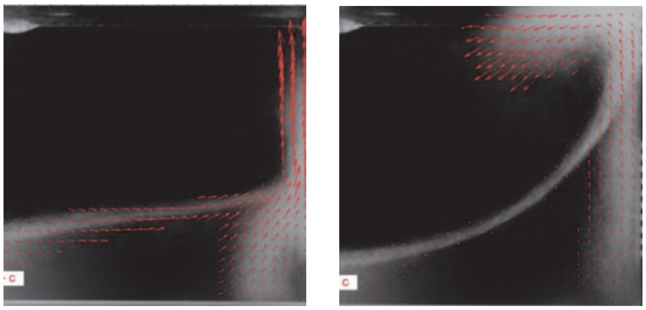
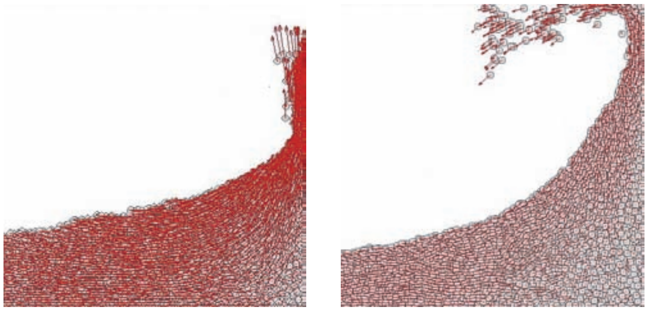


Fig. 4 Enlarged view of the time history of the pressure at point P in one period

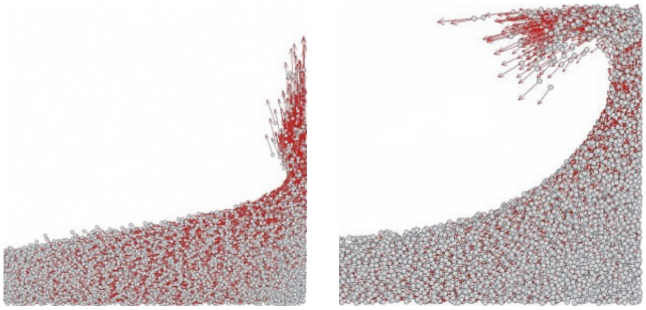
Fig.5 compares some snapshots of free surface deformation and velocity field between experiment, 2D and 3D MPS simulation. An acceptable agreement can be obtained between 2D simulation, 3D simulation and experimental results in terms of both free surface shape and velocity field in sloshing flow on the whole. Thus, the details of free surface deformation, such as the breaking region of the wave and the splash of liquid, can be capsized by 3D MPS method more effectively than 2D MPS method. From the discussion above, we can conclude that the details of flow sloshing, such as breaking wave, overturned free surface and splashing water, can be affected by the dimensional effect in MPS simulation.



(a) experimental results



(b) 2D MPS results



(c) 3D MPS results

Fig. 5 Comparison of snapshots between experiment (upper), 2D MPS simulation (middle) and 3D MPS simulation (lower)

For the sloshing simulation in this paper, 5944 particles and 663458 particles are used in 2D MPS and 3D MPS method respectively, which result in much larger computation time 3D MPS method than 2D MPS method. So 2D MPS method is more efficient to predict quasi-3D sloshing flows in general. However, 3D MPS method is essential to deal with pure 3D sloshing. Furthermore, 3D MPS method is more appropriate than 2D MPS method when smooth pressure filed or sloshing details are required in sloshing problem.

CONCLUSIONS

In this paper, the 3D modified MPS method is utilized to simulate liquid sloshing in rectangular tank under horizontal excitation. Compared the results obtained by 2D and 3D MPS method, the dimension effect has been analyzed. For validation of the present MPS method, a comparison is made between the computational results for 2D and 3D simulation and available experimental data, for which favorable agreement is shown in general. The results show that: 3D MPS method can capsize the details of sloshing flow such as breaking wave and splashing water more efficiently. The results of 3D simulation agree better with experimental results than 2D simulation. For the prediction of impact pressure, 3D MPS method can predict the impact pressure much smoother and more accurately than 2D MPS method. However, because of the less effect of the details, such as breaking wave, overturned free surface and splashing water, computational results between 2D MPS and 3D MPS have an excellent agreement. Considering large computation in 3D simulation, it is sensible for us to apply 2D MPS method to predict sloshing problem generally. Thus, it is essential to utilize 3D simulation in pure 3D flows or other conditions with requirements of smoother pressure field and sloshing details.

ACKNOWLEDGEMENTS

The authors wish to thank Kuang-An Chang and Youn Kyung Song for providing the experimental data of sloshing. This work is supported by National Natural Science Foundation of China (Grant Nos. 51379125, 11272120, 51411130131), the National Key Basic Research Development Plan (973 Plan) Project of China (Grant No. 2013CB036103), High Technology of Marine Research Project of the Ministry of Industry and Information Technology of China, the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning(Grant No. 2013022), Center for HPC of Shanghai Jiao Tong University.

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